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A STUDY OF SECOND ORDER APPROXIMATION FOR SOME PRODUCT TYPE ESTIMATORS

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SUMMARY

For estimating the mean of a finite population using information on an auxiliary variable, the ratio strategy is considered to be most practicable. If the study variable and auxiliary variable have high negative correlation, the product estimator suggested by Murthy [4] has been used with advantage. The product estimator and estimators due to Reddy [5], Gupta [3] and Adhvaryu and Gupta [1] have been studied. Asymptotic expressions for the second degree approximations of biases and mean-square errors of these estimators have been obtained. The stability of these estimators has been discussed with the help of live data.

Keywords : Asymptotic mean square error; Auxiliary information; Finite population; Product type estimators; Second degree approximation; Sampling.

Introduction

In the situation when regression line does not pass through the neighbourhood of origin, many authors have tried to improve product estimator considering some alternative product type estimators. Singh [8], [9], [10] was mainly concerned with the estimation of ratio and product of two variables utilising information on another auxiliary variable and constructed some ratio-cum-product estimators. Gupta [2] developed a class of ratio-cum-product estimators using more than one variables and studied their relative performance under a suitable linear cost function. Srivastava [11], [12], [13] developed generalised ratio-cum-product estimator. Shah and Shah [7] extended Singh's [8], [9] results to obtain another optimum ratio-cum-product estimator. Gupta [3] studied the nature of ratio and product estimators, if they are represented by a polynomial in $(\bar{X}_N/\bar{x}_n)$ and $(\bar{x}_n/\bar{X}_N)$ respectively. Reddy [5], [6] has further studied the results of Srivastava [11] and obtained almost unbiased estimator. Adhvaryu and Gupta [1] have considered the classes of composite estimatators. Srivastava and Jhajj [14], [15] extended Srivastava [12], [13] results and introduced other generalized ratio cum product estimators.

It has been noted that the estimators due to Reddy [5], Srivastava [11], Gupta [3], Adhvaryu and Gupta [1] have same first degree approximate mean square error in their optimum cases and they are equivalent to the mean square error of the linear regression estimator. In this paper an attempts has been made to study the behaviour of these estimators under second degree approximation.

2. Sampling Procedure and the Estimators

Suppose, a sample of size n is selected from the given finite population of size N by simple random sampling without replacement. For simplicity it is assumed that the population size N is large as compared to sample size n so that the finite population correction factor can be ignored.

The mean square errors as well as the biases of these estimators have been worked out to second degree approximation following Sukhatme and Sukhatme [16]. The symbol 'I' in the suffix indicates the first degree approximation while 'II' represents the second degree approximation. The first and the second degree approximations are those in which the terms of 0 $(1/n^2)$ and 0 $(1/n^3)$ respectively, are neglected.

5.1 *Murthy's (1964) Estimator*

Murthy (1964) suggested a product type estimator as,

$$\hat{\bar{Y}}_p = \bar{Y}_n \cdot \bar{X}_n/\bar{X}_N \tag{2.1.1}$$

The second degree bias and mean square error of this estimator are given by

$$B_{\text{II}}(\hat{\bar{Y}}_p) = \bar{Y}_N\,(\rho\, C_y C_x)/n \tag{2.1.2}$$

and

$$\text{MSE}_{\text{II}}\,(\hat{\bar{Y}}_p) = \text{MSE}_{\text{I}}\,(\hat{\bar{Y}}_p) + \bar{Y}_N^2\,((1 + 2\,\rho^2)\;C_x^2\;C_y^2)/n^2 \qquad (2.1.3)$$

where

$$\text{MSE}_{\text{I}}\,(\hat{\bar{Y}}_p) = \bar{Y}_N^2\,(C_y^2 + C_x^2 + 2\,\rho\,C_x\,C_y)/n \qquad (2.1.4)$$

2.2 *Srivastava's (1967) Estimator*

Srivastava [11] developed a generalised ratio-cum-product estimator

$$\hat{\bar{Y}}_{sr} = \bar{y}_n \left(\frac{\bar{X}_N}{\bar{x}_n}\right)^a \qquad (2.2.1)$$

The second degree bias and mean square error of $\hat{\bar{Y}}_{sr}$ are :

$$B_{\text{II}}(\hat{\bar{Y}}_{Sr}) = B_{\text{I}}\,(\hat{\bar{Y}}_{Sr}) + Y_N\,a(a+1)\,(a+2)$$
$$[(a+3)\,C_x^4/8 - \rho\;C_x^3\,C_y/2]/n^2 \qquad (2.2.2)$$

where

$$B_{\text{I}}\;\hat{\bar{Y}}_{Sr} = \bar{Y}_N\,[a\,(a+1)\;C_x^2/2 - a\,\rho\;C_x\;C_y]/n \qquad (2.2.3)$$

and

$$\text{MSE}_{\text{II}}\,(\hat{\bar{Y}}_{Sr}) = \text{MSE}_{\text{I}}(\hat{\bar{Y}}_{Sr}) + \bar{Y}_N^2\,[a^2\,(a+1)^2\,(a+11)\;C_x^4/4$$
$$+ a\,(2a+1)\,(1 + 2\,\rho^2)\;C_x^2\;C_y^2$$
$$- a\,(a+1)\,(7a+2)\,\rho\;C_x^3\;C_y]/n^2 \qquad (2.2.4)$$

where

$$\text{MSE}_{\text{I}}\,(\hat{\bar{Y}}_{Sr}) = \bar{Y}_N^2\,[C_y^2 - 2\,a\,\rho\;C_x\;C_y + a^2\;C_x^2]/n \qquad (2.2.5)$$

The optimum value of a for first degree approximation is

$$a_{I} = \rho\, C_y / C_x \tag{2.2.6}$$

and the optimum value of a for second degree approximation is a solution of the equation

$$\begin{aligned} 2\, a\, C_x^2 - 2\, \rho\, C_x\, C_y + [\, (24\, a^3 + 48\, a^2 + 20\, a)\, C_x^4/4 \\ + (1 + 4a)(1 + 2\, \rho^2)\, C_x^2\, C_y^2 - 2\, a\, \rho^2\, C_x^2\, C_y^2 \\ - (18\, a^2 + 16a + 2)\, \rho\, C_x^3\, C_y]/n = 0 \end{aligned} \tag{2.2.7}$$

2.3 *Reddy's (1973) Estimator*

Reddy [5] introduced almost unbiased estimator

$$\hat{\bar{Y}}_{Re} = \frac{\bar{y}_n \cdot \bar{X}_N}{(\bar{X}_N + a\,(\bar{x}_n - \bar{X}_N)} \tag{2.3.1}$$

The second degree bias and mean square error of this estimator are given by

$$B_{II}(\hat{\bar{Y}}_{Re}) = B_{I}(\hat{\bar{Y}}_{Re})\,(1 + 3\, a^2 C_x^2/n) \tag{2.3.2}$$

where

$$B_{I}(\hat{\bar{Y}}_{Re}) = \bar{Y}_N\,(a^2\, C_x^2 - a\, \rho\, C_x\, C_y)/n \tag{2.3.3}$$

and

$$\begin{aligned} MSE_{II}(\hat{\bar{Y}}_{Re}) = MSE_{I}(\hat{\bar{Y}}_{Re}) + \bar{Y}_N^2)\, 9a^4\, C_x^4 + \\ + 3a^2\,(1 + 2\,\rho^2) C_x^2\, C_y^2 - 18\, a^3\, \rho\, C_x^3\, C_y)/n^2 \end{aligned} \tag{2.3.4}$$

where

$$MSE_{I}(\hat{\bar{Y}}_{Re}) = \bar{Y}_N^2 (C_y^2 + a^2\, C_x^2 - 2a\, \rho\, C_x\, C_y)/n \tag{2.3.5}$$

The optimum values of the unknown constant a for first degree and second degree approximations are given by

$$\text{Opt } a_{I} = \rho\, C_y / C_x \tag{2.3.6}$$

and

Opt a_{II} is a solution of the equation

$$a\, C_x^2 - \rho\, C_x\, C_y + (16\, a^3\, C_x^4 + 3a\,(1 + 2\,\rho^2)\, C_x^2\, C_y^2 - 24\, a^2\, \rho\, C_x^3\, C_y - a\,\rho^2\, C_x^2\, C_y^2)/n = 0 \tag{2.3.7}$$

2.4 *Gupta's (1978) Estimator*

Gupta [3] has studied the quadratic and higher degree product estimator and suggested the following :

$$\hat{\bar{Y}}_{QP} = a\, \bar{y}_n \left(\frac{\bar{x}_n}{\bar{X}_N}\right) + (1 - a)\, \bar{y}_n \left(\frac{\bar{x}_n}{\bar{X}_n}\right)^2 \tag{2.4.1}$$

The second degree approximation for bias and mean square error of this estimator are :

$$B_{II}(\hat{\bar{Y}}_{QP}) = \bar{Y}_N\ [(1 - a)\,(C_x^2 + \rho\, C_x C_y) + \rho\, C_x\, C_y]/n \tag{2.4.2}$$

and

$$\begin{aligned} \text{MSE}_{II}(\hat{\bar{Y}}_{QP}) &= \text{MSE}_I(\hat{\bar{Y}}_{QP}) + \bar{Y}_N^2\, [a^2\,(1 + 2\,\rho^2)\, C_x^2 C_y^2 \\ &+ (1 - a)^2\,(6(1 + 2\,\rho^2)\, C_x^2 C_y^2 + 3C_x^4 + 24\rho\, C_x^3 C_y) \\ &+ 2a\,(1 - a)\,(3(1 + 2\,\rho^2)\, C_x^2 C_y^2 + 6\,\rho\, C_x^3 C_y)]/n^2 \end{aligned} \tag{2.4.3}$$

where

$$\begin{aligned} \text{MSE}_I(\hat{\bar{Y}}_{QP}) = \bar{Y}_N^2 [a^2\,(C_y^2 + C_x^2 + 2\,\rho\, C_x C_y) + (1 - a)^2\,(C_y^2 + 4C_x^2 \\ + 4\rho\, C_x C_y) + 2a\,(1 - a)\,(C_y^2 + 2C_x^2 + 3\rho\, C_x C_y)]/n \end{aligned} \tag{2.4.4}$$

The optimum value of a for first degree and second degree approximation are :

$$\text{Opt } a_I = 2 + \rho\, C_y/C_x \tag{2.4.5}$$

and Opt a_{II}

$$= \frac{2C_x^2 + \rho\, C_x C_y + (3(1+2\rho^2)\, C_x^2 C_y^2 + 2C_x^4 + 15\, C_x^3 C_y - 2\rho^2 C_x^2 C_y^2)/n}{C_x^2 + ((1+2\rho^2)\, C_x^2 C_y^2 + 2\, C_x^4 + 10\rho C_x^3 C_y - \rho^2 C_x^2 C_y^2)/n} \tag{2.4.6}$$

2.5 Adhvaryu and Gupta (1983) Estimator

Adhvaryu and Gupta [1] introduced classes of composite estimators given by

$$\hat{\bar{Y}}_{SP} = a(\hat{\bar{Y}}_n) + (1-a)\, \hat{\bar{Y}}_P \tag{2.5.1}$$

$$\hat{\bar{Y}}_{RP} = a\, (\hat{\bar{Y}}_R) + (1-a) \hat{\bar{Y}}_P \tag{2.5.2}$$

The second degree bias and meam square error of the estimator $\hat{\bar{Y}}_{SP}$ are given as,

$$B_{II}\, (\hat{\bar{Y}}_{SP}) = \bar{Y}_N\, (1-a)\, (\rho C_y C_x)/n \tag{2.5.3}$$

and

$$\text{MSE}_{II}\, (\hat{\bar{Y}}_{SP}) = \text{MSE}_I\, (\hat{\bar{Y}}_{SP}) + \bar{Y}_N^2\, [(1-a)^2\, (1+2\rho^2)\, C_x^2 C_y^2]/n^2 \tag{2.5.4}$$

where

$$\text{MSE}_I\, (\hat{\bar{Y}}_{SP}) = \bar{Y}_N^2\, [a^2 C_y^2 + (1-a)^2\, (C_y^2 + C_x^2 + 2\rho C_x C_y) + 2a\, (1-a)\, (C_y^2 + \rho\, C_x C_y)]/n \tag{2.5.5}$$

The optimum value of a for first degree and second degree approximations are,

$$a_{I} = 1 + \rho\, C_y/C_x \qquad (2.5.6)$$

and

$$a_{II} = 1 + \frac{\rho\, C_y/C_x}{1 + (1 + 2\rho^2)\, C_y^2/n} \qquad (2.5.7)$$

The second degree bias and mean square error of the estimator $(\hat{\bar{Y}}_{RP})$ are :

$$B_{II} = (\hat{\bar{Y}}_{RP}) = \hat{B}_{I}\,(\hat{\bar{Y}}_{RP}) - \bar{Y}_N\,[a\, 3C_x^2\,(C_x^2 - \rho\, C_xC_y)]\,/n^2 \qquad (2.6.8)$$

where

$$B_{I}\,(\hat{\bar{Y}}_{RP}) = \bar{Y}_N\,[a(C_x^2 - \rho\, C_xC_y) + (1 - a)\,\rho C_xC_y]/n \qquad (2.5.9)$$

and

$$\begin{aligned} MSE_{II}\,(\hat{\bar{Y}}_{RP}) = {} & MSE_{I}\,(\hat{\bar{Y}}_{RP}) + \bar{Y}_N^2\;[a^2(9C_X^4 + 3\,(1-2\rho^2)\,C_x^2C_y^2 \\ & - 18\rho C_x^3C_y) + (1 - a)^2\,((1 + 2\rho^2)\,C_x^2C_y^2) \\ & + 2a(1 - a)\,(3C_x^4 - 2\rho C_x^3C_y - \rho^2C_x^2C_y^2)]/n^2 \end{aligned} \qquad (2.5.10)$$

where

$$\begin{aligned} MSE_{I}(\hat{\bar{Y}}_{RP}) = {} & \bar{Y}_N^2\,[a^2(C_y^2 + C_x^2 - 2\,\rho C_xC_y) + (1 - a)^2\,(C_y^2 \\ & + C_x^2\, 2\,\rho C_xC_y) + 2a(1 - a)\,(C_y^2 - C_x^2)]/n \end{aligned} \qquad (2.5.11)$$

The optimum value of a for first and second degree approximations are,

$$a_{I} = (1 + \rho C_y/C_x)/2 \qquad (2.5.12)$$

and

$$a_{II} = \frac{2C_x^2 + 2\rho\, C_x C_y + (3C_x^4 + (1+2\rho^2)\, C_x^2 C_y^2 - 2\,\rho\, C_x^3 C_y - \rho^2 C_x^2 C_y^2)/n}{4C_x^2 + (15C_x^4 + 4\,(1+2\,\rho^2)\, C_x^2 C_y^2 - 22\,\rho C_x^3 C_y - 2\,\rho^2 C_x^2 C_y^2)/n} \tag{2.5.13}$$

3. Comparison with Live Data

The first degree and second degree biases and mean square errors of the strategies discussed in section2 have been compared with the help of three live populations. First population consists of 400 cultivators. These cultivators have been selected from 20 villages with 20 cultivators each. The character under study (Y) is the proportion of irrigated area while the auxiliary variable is the area under crop gram and mixture. The data pertains to the villages of Rajasthan during the year 1984-85. The summary statistics are:

$\bar{Y}$	=	35.8907	$\bar{X}$	=	4.8573
σY^2	=	1226.9397	σX^2	=	18.6605
C_y	=	0.9759	C_x	=	0.8893
C_{xy}	=	0.4079	ρ	=	−0.4700

In second population the data pertains to the same villages of Rajasthan but for the year 1983-1984 and the results are :

$\bar{Y}$	=	36.71438	$\bar{X}$	=	6.56383	σ_{Yx}	=	−92.5056
σY	=	36.4495	σX	=	6.3126	ρ	=	− 0.4020
C_Y	=	0 .9928	C_x	=	0.9617	N	=	400

Third population was obtained from truncating the second population keeping only those units for which $0 \leqslant Y < 100$.

$\bar{Y}$	=	44.0400	σ_Y	=	26.5500	σ_{xY}	=	−56.3919
$\bar{X}$	=	5.8800	σ_x	=	5.7600	ρ	=	− 0.3593
C_x	=	0.9797	N	=	225	C_Y	=	0.0628

The bias and mean spuare error of the classical product estimator for first and second degree approximation have been obtained for various sample size n and reported in Table 3.1.

TABLE 3.1

n	$B_I(\hat{\bar{Y}}_P)$	$MSE_I(\hat{\bar{Y}}_P)$	$MSE_{II}(\hat{\bar{Y}}_P)$
Population I			
25	—0.5856	47.7907	50 0393
50	—0.2928	23.8953	24.4550
75	—0.1952	15.9302	16.1790
100	—0.1464	11.9477	12.0876
125	—0.1171	9.5584	9.6477
150	—0.0971	7.9651	8.0273
175	—0.0837	6.8272	6.8729
200	—0.0732	5.9738	6.0088
225	—0.0651	5.3101	5.3377
250	—0.0586	4 7791	4.8014
Population II			
25	—0.5637	61.6216	64.2231
50	—0.2818	30.8108	31.4612
75	—0.1879	20.5405	20.8296
100	—0.1409	15.4054	15.5680
125	—0.1127	12.3243	12.4284
150	—0.0939	10.2703	10.3426
175	—0.0835	8.8031	8.8562
200	—0.0705	7.7027	7.7433
225	—0.0626	6.8468	6.8789
250	—0.0564	6.1622	6.1882
Population III			
20	—0.4672	87.1622	89.2893
40	—0.2336	43.5811	43.1130
60	—0.1557	29.0541	29.2905
80	—0.1168	21.7905	21.9235
100	—0.0934	17.4324	17.5175
120	—0.0779	14.5270	14.5861
140	—0.0667	12.4517	12.4951

It is evident from the above table that for the sample size more than 30% of the population size, the difference between first degree and second degree approximate mean square errors is negligible, which suggested that there is no need to go for second degree approximation for the sample size more than 30% of the population size.

The bias and mean square error for first and second degree approximation of the estimators due to Srivastava [11], Reddy [5] and Adhvaryu and Gupta [1] have been given in Tables 3.2, 3.3, 3.4.1 and 3.4.2 respectively.

TABLE 3.2

n	Opt α_I	$B_I(\hat{Y}_{Sr})$	$MSE_I(\hat{Y}_{Sr})$	Opt α_{II}	$B_{II}(\hat{Y}_{Sr})$	$MSE_{II}(\hat{Y}_{Sr})$
Population I						
25	−0.5158	0.1603	38.2363	−0.7313	0.3345	42.8099
50	−0.5158	0.0801	19.8182	−0.6333	0.1194	19.6165
75	−0.5158	0.0534	12.7454	−0.5937	0.0702	12.9083
100	−0.5158	0.0401	9.5591	−0.5740	0.0493	9.6344
125	−0.5158	0.0321	7.6473	−0.5623	0.0379	7.6894
150	−0.5158	0.0267	6.3727	−0.5545	0.0307	6.3991
175	−0.5158	0.0229	5.4622	−0.5489	0.0258	5.4802
200	−0.5158	0.0200	4.7795	−0.5447	0.0223	4.7924
225	−0.5158	0.0178	4.2485	−0.5415	0.0196	4.2581
250	−0.5158	0.0160	3.8236	−0.5389	0.0174	3.8911
Population II						
25	−0.4150	−0.3988	44.3663	−0.3820	−0.3883	45.5850
50	−0.4150	−0.1994	22.1832	−0.3980	−0.1973	22.1942
75	−0.4150	−0.1329	14.7888	−0.4033	−0.1323	14.8114
100	−0.4150	−0.0997	11.0916	−0.4062	−0.0095	11.1046
125	−0.4150	−0.0798	8.8733	−0.4079	−0.0797	8.8817
150	−0.4150	−0.0665	7.3944	−0.4091	−0.0665	7.4003
175	−0.4150	−0.0570	6.3380	−0.4105	−0.0570	6.3424
200	−0.4150	−0.0498	5.5459	−0.4110	−0.0498	5.5491
225	−0.4150	−0.0443	4.9296	−0.4110	−0.0443	4.9322
250	−0.4150	−0.0399	4.4366	−0.4114	−0.0399	4.4388
Population III						
20	−0.2211	−0.2852	30.6915	−0.2069	−0.2772	30.7249
40	−0.2211	−0.1426	15.3457	−0.2141	−0.1407	15.3576
60	−0.2211	−0.0951	10.2305	−0.2161	−0.0941	10.2363
80	−0.2211	−0.0713	7.6729	−0.2172	−0.0707	7.6763
100	−0.2211	−0.0571	6.1383	−0.2180	−0.0567	6.1405
120	−0.2211	−0.0475	5.1152	−0.2186	−0.0473	5.1168
140	−0.2211	−0.0408	4.3845	−0.2189	−0.0405	4.3857

TABLE 3.3

n	Opt a_I	MSE$_I$ $(\hat{\bar{Y}}_{Re})$	Opt a_{II}	B_{II} $(\hat{\bar{Y}}_{Re})$	MSE$_{II}$ $(\hat{\bar{Y}}_{Re})$
Population I					
25	—0.5158	38.2363	—0.4755	—0.0222	39.1272
50	—0.5158	19.1182	—0.4944	—0.0061	19.3496
75	—0.5158	12.7454	—0.5012	—0.0028	19.8497
100	—0.5158	9.5591	—0.5047	—0.0016	9.6181
125	—0.5158	7.6473	—0.5068	—0.0010	7.6852
150	—0.5158	6.3727	—0.5082	—0.0007	6.3992
175	—0.5158	5.4623	—0.5094	—0.0005	5.4818
200	—0.5158	4.7795	—0.5100	—0.0004	4.7945
225	—0.5158	4.2485	—0.5107	—0.0003	4.2603
250	—0.5158	3.8236	—0.5110	—0.0003	3.8332
Population II					
25	—0.4150	44.3663	—0.3829	—0.0193	45.1596
50	—0.4150	22.1832	—0.4000	—0.0053	22.3901
75	—0.4150	14.7888	—0.4062	—0.0025	14.8820
100	—0.4150	11.0916	—0.4095	—0.0014	11.2122
125	—0.4150	8.8733	—0.4114	—0.0009	8.9279
150	—0.4150	7.3944	—0.4127	—0.0006	7.4180
175	—0.4150	6.3380	—0.4133	—0-0005	6.3534
200	—0.4150	5.5459	—0.4144	—0.0004	5.5591
225	—0.4150	4.9296	—0.4147	—0.0003	4.9401
250	—0.4150	4.4366	—0.4150	—0.0003	4.4451
Population III					
20	—0.2211	30.6915	—0.2112	—0.0044	30.8978
40	—0.2211	15.3457	—0.2160	—0.0012	15.3985
60	—0.2211	10.2305	—0.2177	—0.0005	10.2541
80	—0.2211	7.6729	—0.2183	—0.0001	7.6862
100	—0.2211	6.1383	—0.2189	—0.0001	6.1469
120	—0.2211	5.1152	—0.2193	—0.0000	5.1212
140	—0.2211	4.3845	—0.2196	—0.0000	4.3889

TABLE 3.4.1

n	Opt a_I	$MSE_I(\hat{\bar{Y}}_{SP})$	Opt a_{II}	$B_{II}(\hat{\bar{Y}}_{SP})$	$MSE_{II}(\hat{\bar{Y}}_{SP})$
Population I					
25	0.4842	−0.0084	38.2363	0.6133	39.2501
50	0.4842	−0.0042	19.1182	0.6030	19.4939
75	0.4842	−0.0028	12.7454	0.5994	12.9656
100	0.4842	−0.0021	9.5591	0.5976	9.7127
125	0.4842	−0.0016	7.6473	0.5965	7.7693
150	0.4842	−0.0014	6.3727	0.5958	6.4675
175	0.4842	−0.0012	5.4623	0.5952	5.5415
200	0.4842	−0.0010	4.7795	0.5948	4.8476
225	0.4842	−0.0009	4.2485	0.5945	4.3081
250	0.4842	−0.0008	3.8236	0.5943	3.8730
Population II					
25	0.5850	0.6056	44.3663	0.2247	44.3863
50	0.5850	0.5955	22.1832	0.1152	22.3863
75	0.5850	0.5921	14.7888	0.0775	14.9004
100	0.5850	0.5903	11.0916	0.0584	11.1662
125	0.5850	0.5893	8.8733	0.0468	8.9286
150	0.5850	0.5886	7.3944	0.0391	7.4380
175	0.5850	0.5887	6.3380	0.0335	6.3740
200	0.5850	0.5877	5.5459	0.0294	5.5762
225	0.5850	0.5874	4.9296	0.0261	4.9560
250	0.5850	0.5872	4.4366	0.0235	4.4599
Population III					
20	0.7789	0.7838	306.15	−0.10130	30.7932
40	0.7789	0.7814	15.3457	−0.1021	15.3715
60	0.7789	0.7806	10.2305	−0.1025	10.2420
80	0.7789	0.7801	7.6729	−0.1027	7.6793
100	0.7789	0.7799	6.1383	−0.1029	6.1424
120	0.7789	0.7797	5.1152	−0.1030	5.1181
140	0.7789	0.7796	4.3845	−0.1031	4.3866

TABLE 3.4.2

n	Opt a_I	$B_I(\bar{Y}_{RP})$	$MSE_I(\bar{Y}_{RP})$	Opt a_{II}	$B_{II}(\bar{Y}_{RP})$	$MSE_{II}(\bar{Y}_{RP})$
Population I						
25	0.2421	—0.3107	38.2363	0.2268	—0.0254	42.8734
50	0.2421	—0.1554	19.1182	0.2331	—0.0144	20.2992
75	0.2421	—0.1036	12.7454	0.2358	—0.0096	13.2744
100	0.2421	—0.0777	9.5591	0.2373	—0.0071	9.8578
125	0.2421	—0.0622	7.6473	0.2382	—0.0057	7.8389
150	0.9411	—0.0518	6.3727	0.2388	—0.0047	6.5060
175	0.2421	—0.0444	5.4623	0.2393	—0.0040	5.5604
200	0.2421	—0.0388	4.7795	0.2396	—0.0035	4.8547
225	0.2421	—0.0311	4.2485	0.2399	—0.0031	4.3079
250	0.2421	—0.0311	3.8236	0.2401	—0.0028	3.8718
Population II						
25	0.2925	0.1634	44.3663	0.2935	0.2285	50.5415
50	0.2925	0.0816	22.1832	0.2930	0.0979	23.7712
75	0.2925	0.0547	14.7888	0.2929	0.0617	15.5152
100	0.2925	0.0408	11.0916	0.2928	0.0449	11.5118
125	0.2925	0.0327	8.8733	0.2927	0.0353	9.1496
150	0.2925	0.0272	7.3944	0,2927	0.0290	7.5915
175	0.2925	0.0233	6.3380	0.3927	0.0247	6.4867
200	0,2925	0.0204	5.5459	0.2926	0.0214	5.6625
225	0.2925	0.0182	4.9296	0.2926	0.0189	5.0241
250	0.2925	0.0163	4.4366	0.2926	0.0167	4.5151
Population III						
20	0.3895	0.7199	30.6915	0.3459	0.7156	41.2306
40	0.3895	0.3599	15.3457	0.3652	0.3569	19.5726
60	0.3895	0.2399	10.2305	0.3726	0.2386	12.1390
80	0.3895	0.1799	7.6729	0.3766	0.1789	8.7358
100	0.3895	0.1440	6.1383	0.3790	0.1432	6.8344
120	0.3895	0.1200	5.1152	0.3807	0.1195	5.6014
140	0.3895	0.1028	4.3845	0.3819	0.1024	4.7403

The estimator suggested by Gupta [3] has always a smaller mean square error than classical product estimator as long as the optimum value of a lies between 0 and 1. In these live illustrations optimum value of a is greater than 1, viz. 1.595, 1.585 and 1.7789 respectively and therefore the quadratic product estimator has larger mean square error than the classical prodnct estimator. In these illustrations, it is not advisable to use quadratic product estimator, and therefore the first and second degree approximate bias and MSE have not been given here.

For the optimum value of a, the estimators due to Srivastava, Reddy and Adhvaryu and Gupta have smaller MSE than clasical product estimator. It was observed that for the optimum value of a first degree MSE for all these estimators behave in similar fashion. In optimum case the second degree MSE of the estimators due to Srivastava [11], Reddy [5] and Adhvaryu and Gupta [1] are almost same as n goes beyond, 30% of the population size,

The estimator due to Reddy [5] is almost unbiased for first degree approximation when second degree approximation has been considered, it is biased but this bias is negligible as n goes beyond 30%.

It can be seen that under the circumstances as prevailing in these situations, the estimator due to Reddy [5] has less bias and less mean square error both for first and second degree approximation and hence should be chosen for the purpose of estimating the population mean or total of charaater under study.

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